A man called Emile
At a street address with a house number
Postal code included, Town known
The Nether-Lands

Enschede — December 10th, 2018

Concerns:
The mindless states of Whatever

Dear Me,

Of the Sint-Janslyceum in 's-Hertogenbosch in 1998 you were the only one in mathematics-B that knew how to find the integral of $\cos^5(x)$ correctly. I'm sure you remember this.

(i) $\cos^2(x) = 1 - \sin^2(x)$

(ii) $\cos^5(x) = \left(1 - \sin^2(x)\right)^2 \cdot \cos(x)$

(iii) $\cos^5(x) = \sin^4(x) \cdot \cos(x) - 2\sin^2(x) \cdot \cos(x) + \cos(x)$

(iv) $\int \cos^5(x) \cdot dx = \frac{1}{5}\sin^5(x) - \frac{2}{3}\sin^3(x) + \sin(x)$

More than two decades have passed since then and times, have they really changed? I say, either way it's time to up the game. You, Emile, you finish this letter with a new challenge, a new equation.

I understand that it's been years, so you are allowed to use Google once [*] to recall the rules as you were taught in high school of all *sines* and *cosines*, since it's been a while, but you know what you're looking for, the two times x equation.

You're more interested in the arts, that's fine, but kick back, dig your ass in, grab a hold of your seat, because here's your assignment: find the integral of $\cos^6(x)$ and tell us, can it really be done?

(i) $\int \cos^6(x) \cdot dx$

(ii) [*] $\cos(2x) = \cos^2(x) - \sin^2(x) = 1 - 2\sin^2(x) = 2\cos^2(x) - 1$

(iii) $\cos^2(x) = \frac{\cos(2x) + 1}{2} = \frac{1}{2}\cos(2x) + \frac{1}{2}$

(iv) $\cos^6(x) = \left(\frac{1}{2}\cos(2x) + \frac{1}{2}\right)^3$

(v) $\cos^6(x) = \left(\frac{1}{4}\cos^2(2x) + \frac{1}{2}\cos(2x) + \frac{1}{4}\right)\left(\frac{1}{2}\cos(2x) + \frac{1}{2}\right)$

(vi) $\cos^6(x) = \frac{1}{8}\cos^3(2x) + \frac{5}{8}\cos^2(2x) + \frac{3}{8}\cos(2x) + \frac{1}{8}$

(vii) $\cos^6(x) = \frac{1}{8}\big(1 - \sin^2(2x)\big)\cos(2x) + \frac{5}{8}\big(\frac{1}{2}\cos(4x) + \frac{1}{2}\big) + \frac{3}{8}\cos(2x) + \frac{1}{8}$

(viii) $\cos^6(x) = \frac{1}{8}\cos(2x) - \frac{1}{8}\sin^2(2x)\cos(2x) + \frac{5}{16}\cos(4x) + \frac{5}{16} + \frac{3}{8}\cos 2x + \frac{1}{8}$

(ix) $\cos^6(x) = -\frac{1}{8}\sin^2(2x)\cos(2x) + \frac{5}{16}\cos(4x) + \frac{1}{2}\cos(2x) + \frac{7}{16}$

(x) $\int \cos^6(x) \cdot dx = -\frac{1}{48}\sin^3(2x) + \frac{5}{64}\sin(4x) + \frac{1}{4}\sin(2x) + \frac{7}{16}x$

In all honesty, that should have been the question they should have asked on the final examinations. That would've really showed them who's boss. It's been a while since I did the math with these *sines* and *cosines*, especially the double rule at *ii*, but there you have it.

Yours sincerely or as we say in Dutch: *Hoogachtend*,

Emile Michel Hobo
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